{UNIT - II}

Fourier Transform :-

It is an extension from Fourier series.

An aperiodic signal can be generated from a periodic signal by extending the period to infinity.

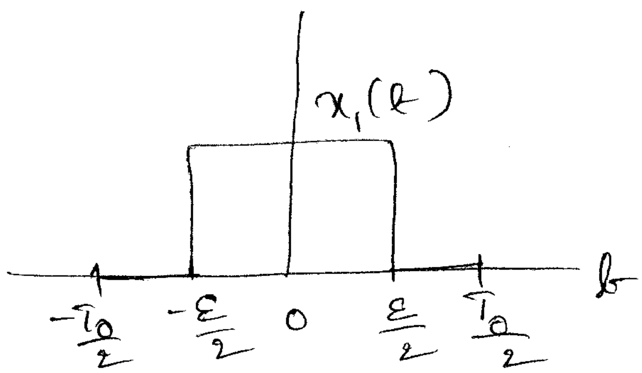
Let $x_1(t) = \begin{cases} 0, & -\frac{T_0}{2} \leq t < -\frac{\varepsilon}{2} \\ 1, & -\frac{\varepsilon}{2} \leq t < \frac{\varepsilon}{2} \\ 0, & \frac{\varepsilon}{2} \leq t < \frac{T_0}{2} \end{cases}$

Let $x_{T_0}(t)$ be the periodic extension of the above signal.

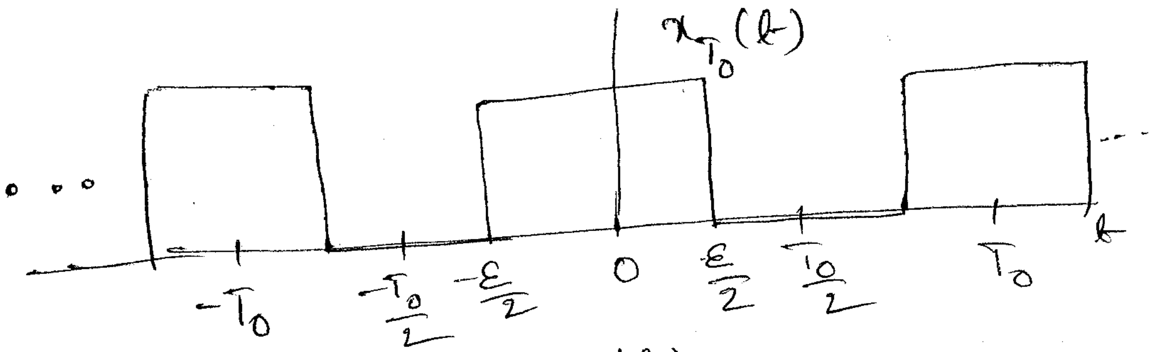
As $T_0 \to \infty$, the periodic signal will be reduced to aperiodic signal.

i.e $x(t) = \underset{T_0 \to \infty}{Lt}\, x_{T_0}(t)$

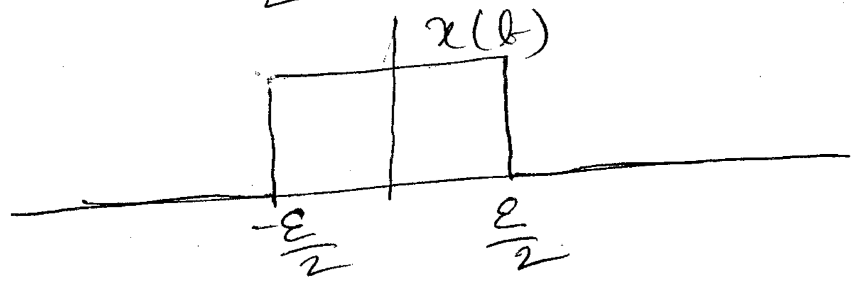
(Interval of definition)



(Periodic extension)



(Aperiodic signal)



The periodic signal $x_{T_0}(t)$ has an exponential fourier series

$$x_{T_0}(t) = \sum_{m=-\infty}^{\infty} X_m e^{jm\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T_0}.$$

where

$$X_m = \frac{1}{T_0} \int_{T_0} x_{T_0}(t) e^{-jm\omega_0 t} dt.$$

$$X_m = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} x_{T_0}(t) e^{-jm\omega_0 t} dt$$

$$= \frac{1}{T_0} \int_{-\infty}^{\infty} x(t) e^{-jm\omega_0 t} dt.$$

Let $$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

then $X_m = \frac{1}{T_0} X(m\omega_0)$

$$x_{T_0}(t) = \sum_{m=-\infty}^{\infty} \frac{1}{T_0} X(m\omega_0) e^{jm\omega_0 t}$$

Let $\Delta\omega = \frac{2\pi}{T_0}$

$$x_{T_0}(t) = \frac{2\pi}{2\pi} \sum_{m=-\infty}^{\infty} \frac{1}{T_0} X(m\omega_0) e^{jm\omega_0 t}$$

$$= \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} X(m\omega_0) e^{jm\omega_0 t} \Delta\omega$$

$$x(t) = \lim_{T_0\to\infty} x_{T_0}(t) = \lim_{T_0\to\infty} \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} X(m\omega_0) e^{jm\omega_0 t} \Delta\omega$$

As $m\omega_0 \to \omega$, $\Delta\omega \to d\omega$ and the summation goes over to an integral.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega.$$

Definition:-

Let $x(t)$ be a signal such that:

(a) $x(t)$, $-\infty < t < \infty$

(b) $\int_{-\infty}^{\infty} |x(t)|\,dt \leq M < \infty$, for $0 < M < \infty$

Then the Fourier transform of $x(t)$ is given as

$$X(\omega) = \cancel{\tfrac{1}{2\pi}}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \mathcal{F}\{x(t)\}$$

The Inverse Fourier transform is defined as

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\, d\omega = \mathcal{F}^{-1}\{X(\omega)\}.$$

There is a one-to-one relationship between a signal and its Fourier transform.

Definition:- The pair

$$\mathcal{F}\{x(t)\} = X(\omega) \leftrightarrow x(t) = \mathcal{F}^{-1}\{X(\omega)\}$$

is called a Fourier transform pair.

The Fourier transform can be expressed using frequency in terms of hertz

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$\omega = 2\pi f$

$d\omega = 2\pi\, df.$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

eg:- Consider the pulse

$$x(t) = \Pi\left(\frac{t}{\varepsilon}\right)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\frac{\varepsilon}{2}}^{\varepsilon/2} 1\, e^{-j\omega t}\, dt = -\frac{e^{-j\omega t}}{j\omega}\Bigg|_{-\frac{\varepsilon}{2}}^{\frac{\varepsilon}{2}}$$

$$= \frac{1}{j\omega}\, \frac{e^{\frac{j\omega\varepsilon}{2}} - e^{-\frac{j\omega\varepsilon}{2}}}{2j} \times 2j$$

$$= \frac{\cancel{2}}{\omega}\, \frac{\sin\left(\frac{\omega\varepsilon}{2}\right)}{\frac{\omega\varepsilon}{2}} \times \frac{\cancel{\omega}\varepsilon}{\cancel{2}} = \varepsilon\, Sa\left(\frac{\omega\varepsilon}{2}\right)$$

$$F\left\{\Pi\left(\frac{t}{\varepsilon}\right)\right\} = \varepsilon\, Sa\left(\frac{\omega\varepsilon}{2}\right) = X(\omega).$$

Def:- The signal $x(t)$ is piece-wise smooth if it can be divided into a finite number of intervals $a_i < t < b_i$ such that $x(t)$ has continuous derivatives on these intervals.

Suppose $x(t)$ is piece-wise smooth and it satisfies

$$\int_{-\infty}^{\infty} |x(t)|\, dt \le M < \infty$$

Then $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

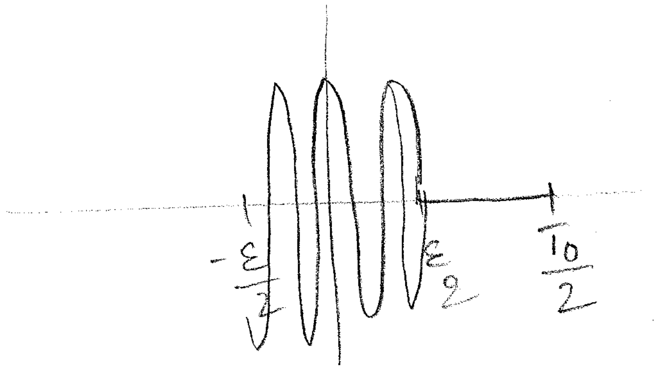
exists and it is a continuous function of ω.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$ exists and the

equality holds at every point where $x(t)$ is continuous.

If $x(t)$ is discontinuous at $t = t_0$, then the integral converges to the average of the left and right-hand limits.

$$x_c(t) = \begin{cases} A\cos(\omega_c t) & -\frac{\varepsilon}{2} \le t \le \frac{\varepsilon}{2} \\ 0 & \frac{\varepsilon}{2} < t < T_0 - \frac{\varepsilon}{2} \end{cases}$$



Properties of Fourier Transform:-

Assume there exists a Fourier transform for the signals $x(t)$ and $h(t)$.

1. Linearity:-

The Fourier transform satisfies

$$F\{a_1 x_1(t) + a_2 x_2(t)\} = a_1 X_1(\omega) + a_2 X_2(\omega)$$

2. Duality:-

If $x(t) \leftrightarrow X(\omega)$ is a Fourier transform pair, then $X(t) \leftrightarrow 2\pi\, x(-\omega)$ is also a Fourier transform pair.

eg:- Consider the Fourier transform pair

$$\Pi\left(\frac{t}{\varepsilon}\right) \leftrightarrow \varepsilon\, Sa\left(\frac{\omega\varepsilon}{2}\right)$$

$$\varepsilon\, Sa\left(\frac{\varepsilon t}{2}\right) \leftrightarrow 2\pi\,\Pi\left(\frac{-\omega}{\varepsilon}\right) = 2\pi\,\Pi\left(\frac{\omega}{\varepsilon}\right)$$ is also a Fourier transform pair.

3. Parseval's Theorem :-

Let $x(t)$ and $y(t)$ be signals with a Fourier transform,

Then $\int_{-\infty}^{\infty} x(t)\,\bar{y}(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,\bar{Y}(\omega)\,d\omega.$

when $x(t) = y(t)$

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega.$$

4. Time and Frequency scaling

For any +ve number a, we have

$$F\{x(at)\} = \frac{1}{a} X\left(\frac{\omega}{a}\right)$$

eg:- Consider the Fourier Transform pair and the signal $\hat{x}(t) = \Pi\left(\frac{t}{\frac{\varepsilon}{2}}\right) = \Pi\left(\frac{2t}{\varepsilon}\right)$

$$\Pi\left(\frac{t}{\varepsilon}\right) \leftrightarrow \varepsilon\, Sa\left(\frac{\omega\varepsilon}{2}\right)$$

$$F\{\hat{x}(t)\} = F\left\{\Pi\left(\frac{2t}{\varepsilon}\right)\right\} = \frac{2}{\text{[illegible]}}\left(\frac{\varepsilon}{\text{[illegible]}}\right) Sa\left(\frac{\varepsilon\omega}{2}\right)$$

$$F\{ax(t)\} \leftrightarrow a\,X(\omega)$$

$$F\{x(at)\} = \frac{1}{a} X\left(\frac{\omega}{a}\right)$$

$$F\left\{x\left(2\frac{t}{\varepsilon}\right)\right\} = \frac{\varepsilon}{2}\, Sa\left(\frac{\omega\varepsilon}{2\times 2}\right) = \frac{\varepsilon}{2}\, Sa\left(\frac{\omega\varepsilon}{4}\right)$$

5. Convolution in the Time Domain :-

If $x(t)$ and $h(t)$ are Fourier transformable,

$$F\left\{ \int_{-\infty}^{\infty} h(t-\lambda)\, x(\lambda)\, d\lambda \right\} = H(\omega)\, X(\omega).$$

Consider $\Pi\left(\frac{t}{\mathcal{E}}\right) \leftrightarrow \frac{\mathcal{E}}{2} Sa\left(\frac{\omega \mathcal{E}}{4}\right)$

Let $h(t) = \Pi\left(\frac{2t}{\mathcal{E}}\right)$

$$h(t-\lambda) = \Pi\left(\frac{t-\lambda}{\mathcal{E}/2}\right)$$

$$F\left\{ \int_{-\infty}^{\infty} \Pi\left(\frac{t-\lambda}{\mathcal{E}/2}\right) \Pi\left(\frac{\lambda}{\mathcal{E}/2}\right) d\lambda \right\} = \left(\frac{\mathcal{E}}{2}\right)^2 Sa^2\left(\frac{\omega \mathcal{E}}{4}\right) \quad -①$$

but $\int_{-\infty}^{\infty} \Pi\left(\frac{t-\lambda}{\mathcal{E}/2}\right) \Pi\left(\frac{\lambda}{\mathcal{E}/2}\right) d\lambda = \frac{\mathcal{E}}{2} \Lambda\left(\frac{t}{\mathcal{E}}\right) \quad -②$

From one and ②

$$\frac{\mathcal{E}}{2} \Lambda\left(\frac{t}{\mathcal{E}}\right) \leftrightarrow \left(\frac{\mathcal{E}}{2}\right)^2 Sa^2\left(\frac{\omega \mathcal{E}}{4}\right)$$

$$\Lambda\left(\frac{t}{\mathcal{E}}\right) \leftrightarrow \left(\frac{\mathcal{E}}{2}\right) Sa^2\left(\frac{\omega \mathcal{E}}{4}\right)$$ is a Fourier Transform pair.

6. Convolution in the frequency domain :-

Let $x(t)$ and $h(t)$ be two signals,

$$F\{ h(t)\, x(t) \} = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega-\lambda)\, X(\lambda)\, d\lambda$$

7. Differentiation:-

If $x(t)$ has an nth derivative, then

$$F\{x^{(n)}(t)\} = (j\omega)^n X(\omega)$$

8. Integration:-

For the signal $x(t)$, we have

$$F\left\{\int_{-\infty}^{t} x(\lambda)\, d\lambda\right\} = \frac{1}{j\omega} X(\omega) + \pi X(0)\,\delta(\omega)$$

9. Time shift :-

$$F\{x(t-t_0)\} = e^{-j\omega t_0} X(\omega)$$

10. Frequency shift :-

$$F\{e^{j\omega_0 t} x(t)\} = X(\omega-\omega_0)$$

11. Modulation :-

$$F\{x(t)\cos(\omega_0 t)\} = \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$$

↑ multiplied by a cosine wave

$$F\{x(t)\sin(\omega_0 t)\} = \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$$

eg:- Consider the signal

$$x(t) = \Pi\left(\frac{t}{\varepsilon}\right) \cos(\omega_0 t)$$

To find the Fourier transform of this signal

$$\rightarrow \Pi\left(\frac{t}{\varepsilon}\right) \leftrightarrow \varepsilon\, Sa\left(\frac{\omega\varepsilon}{2}\right)$$

$$F\left\{\Pi\left(\frac{t}{\varepsilon}\right)\cos(\omega_0 t)\right\} = \frac{1}{2}\left[\varepsilon\, Sa\left(\frac{(\omega-\omega_0)\varepsilon}{2}\right) + \varepsilon\, Sa\left(\frac{(\omega+\omega_0)\varepsilon}{2}\right)\right]$$

$$= \frac{\varepsilon}{2}\left[Sa\left(\frac{\varepsilon(\omega-\omega_0)}{2}\right) + Sa\left(\frac{\varepsilon(\omega+\omega_0)}{2}\right)\right]$$

Find F.T of

(a) $\Pi\left(\frac{2t}{\varepsilon}\right)$

(b) $\Lambda\left(\frac{t}{\varepsilon}\right)$ where $\int_{-\infty}^{\infty} \Pi\left(\frac{t-\lambda}{\varepsilon/2}\right)\Pi\left(\frac{\lambda}{\varepsilon/2}\right) d\lambda = \frac{\varepsilon}{2}\Lambda\left(\frac{t}{\varepsilon}\right)$

(c) $\delta(t)$ where $\delta(t) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon}\Pi\left(\frac{t}{\varepsilon}\right)$ (Limits of signals)

(d) 1

(e) $\cos(\omega_0 t)$ (f) $e^{jm\omega_0 t}$

(g) $x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jm\omega_0 t}$

Generalized Fourier Transform:-

$$\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \Pi\left(\frac{t}{\varepsilon}\right) = \delta(t)$$

The F.T of the pulse function is

$$F\left\{\frac{1}{\varepsilon}\Pi\left(\frac{t}{\varepsilon}\right)\right\} = \frac{1}{\varepsilon}\,\varepsilon\, Sa\left(\frac{\omega\varepsilon}{2}\right) = Sa\left(\frac{\omega\varepsilon}{2}\right)$$

As the pulse becomes narrower in time, The F.T of the pulse is

$$\lim_{\varepsilon \to 0}\left\{\frac{1}{\varepsilon}\Pi\left(\frac{t}{\varepsilon}\right)\right\} = \lim_{\varepsilon \to 0} Sa\left(\frac{\omega\varepsilon}{2}\right) = 1$$

Def:- The generalized F.T of the impulse function is the constant function.

$$x(t) = \delta(t) \leftrightarrow 1 = x(t)$$

eg:- $x(t) = 1$ $\qquad \delta(t) \leftrightarrow 1$

$$1 \leftrightarrow 2\pi\,\delta(\omega)$$

Applying modulation property

$$F\{1 \cos(\omega_0 t)\} = 2\pi\left\{\frac{1}{2}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]\right\}$$

$$= \pi\left\{\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]\right\}$$

eg:- F.T of Fourier series

$$x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jm\omega_0 t}$$

Fourier transform pairs:-

Signal	F.T
$\Pi\left(\frac{t}{\varepsilon}\right)$	$\varepsilon Sa\left(\frac{\omega\varepsilon}{2}\right)$
$\varepsilon Sa\left(\frac{\varepsilon t}{2}\right)$	$2\pi\, \Pi\left(\frac{\omega}{\varepsilon}\right)$
$\Lambda\left(\frac{t}{\varepsilon}\right)$	$\frac{\varepsilon}{2} Sa^2\left(\frac{\omega\varepsilon}{4}\right)$
$\frac{\varepsilon}{2} Sa^2\left(\frac{t\varepsilon}{4}\right)$	$2\pi\, \Lambda\left(\frac{\omega}{\varepsilon}\right)$
$\delta(t)$	1
1	$2\pi\,\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\,\delta(\omega-\omega_0)$
$\cos(\omega_0 t)$	$\pi\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$
$\sin(\omega_0 t)$	$j\pi\left[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right]$
$u_s(t)$	$\frac{1}{j\omega}+\pi\,\delta(\omega)$
$\mathrm{Sgn}(t)$	$\frac{2}{j\omega}$
$e^{-at}u_s(t),\ a>0$	$\frac{1}{a+j\omega}$
$\frac{t^n}{n!}e^{-at}u_s(t),\ a>0$	$\frac{1}{(a+j\omega)^{n+1}}$
$e^{-a\lvert t\rvert}\ \ a>0$	$\frac{2a}{a^2+\omega^2}$
$e^{-\frac{t^2}{2\sigma^2}}$	$(\sigma\sqrt{2\pi})\, e^{-\frac{\sigma^2\omega^2}{2}}$

Laplace Transform:-

Let $x(t)$ be a signal. The L.T is a mathematical tool that transforms this signal representation (time-domain) into a completely different signal representation. (a fn. of a complex variable s).

Not all signals are Laplace transformable.

Def:- Let $x(t)$ $-\infty < t < \infty$, be a signal that satisfies

(a) $x(t) = 0$ for $t < 0$,

(b) $\int_0^\infty |x(t)| e^{-\sigma t} dt < \infty$ for $0 \le \sigma_0 < \sigma < \infty$

Then the laplace transform of $x(t)$ is defined as

$$L\{x(t)\} = \int_0^\infty x(t) e^{-st} dt = X(s).$$

The inverse L.T of $X(s)$ is given by

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} X(s) e^{st} ds = L^{-1}\{X(s)\}.$$

's' is called the Laplace variable.

The pair $L\{x(t)\} = X(s) \leftrightarrow x(t) = L^{-1}\{X(s)\}$

is called a Laplace Transform pair.

eg.- Consider the signals (a) $\sin(\omega t)$, $-\infty < t < \infty$ ×

(b) $[\sin(\omega t) \cos(t)]$, $-\infty < t < \infty$ ✓

Existence of the L.T:-

(i) $\sin(\omega t)$, $-\infty < t < \infty$

The L.T does not exist since $x(t) \neq 0$ for $t<0$.

(ii) $\sin(\omega t)\, u_s(t)$, $-\infty < t < \infty$

$x(t) = \sin(\omega t)$, $t > 0$.

$$\cancel{X(s)=}\left| \int_0^\infty \sin(\omega t)\, e^{-st}\, dt \right| < \infty$$

$$\left| \int_0^\infty \sin(\omega t)\, e^{-st}\, dt \right| \le \int_0^\infty \left| \sin(\omega t)\, e^{-st} \right| dt \le \int_0^\infty |\sin(\omega t)|\, |e^{-st}|\, dt$$

Let $s = \sigma + j\omega$ with $\sigma \ge 0$, then

$$|e^{-st}| = |e^{-(\sigma+j\omega)t}| = e^{-\sigma t}\, |e^{-j\omega t}| = e^{-\sigma t}$$

$$\therefore \left| \int_0^\infty x(t)\, e^{-st}\, dt \right| \le \int_0^\infty |x(t)|\, e^{-\sigma t}\, dt < \infty \quad \text{— (b)}$$

The defining integral of the L.P will exist when (b) is satisfied.

<u>Problem</u>:-

$$x(t) = e^{at}\, u_s(t)$$

$$\int_0^\infty |e^{at}\, u_s(t)|\, e^{-\sigma t}\, dt = \lim_{\gamma\to\infty} \int_0^\gamma |e^{at} u_s(t)|\, e^{-\sigma t}\, dt$$

$$= \lim_{\gamma\to\infty} \left. \frac{e^{at}\, e^{-\sigma t}}{(a-\sigma)} \right|_0^\gamma = \lim_{\gamma\to\infty} \frac{e^{(a-\sigma)\gamma} - 1}{a-\sigma} = \frac{-1}{a-\sigma}, \quad \sigma > a$$

The integral is finite if $\sigma > \sigma_0 \geq a$

$$X(s) = \int_0^{\infty} x(t)\, e^{-st}\, dt = \underset{\gamma\to\infty}{Lt} \int_0^{\gamma} e^{at} u_s(t) e^{-st}\, dt$$

$$= \underset{\gamma\to\infty}{Lt} \left. \frac{e^{(a-s)t} - 1}{a-s} \right|_0^{\gamma} = \underset{\gamma\to\infty}{Lt} \frac{e^{(a-s)\gamma} - 1 - \gamma + \gamma}{a-s}$$

$$= \frac{-1}{a-s}, \quad s > a \text{ i.e } (\sigma > a)$$

$$= \frac{1}{s-a}$$

$$\therefore L\{e^{at} u_s(t)\} = \frac{1}{s-a}$$

As $a \to 0$

$$L\{u_s(t)\} = \frac{1}{s}$$

Problem 2:- Let $x(t) = \delta(t)$

$$X(s) = \int_{0^-}^{\infty} \delta(t)\, e^{-st}\, dt = 1$$

Region of Convergence:-

The set of all complex numbers for which the defining integral of the L.T exists is called the region of Convergence. (ROC).

Signal	L.T	ROC
$\delta(t)$	1	$s > -\infty$ (entire s-plane).
$u_s(t)$	$\frac{1}{s}$	$s > 0$
$e^{at}\, u_s(t)$	$\frac{1}{s-a}$	$s > a$
$e^{-at}\, u_s(t)$	$\frac{1}{s+a}$	$s > -a$

Problem 3:- $\quad x(t) = \cos(\omega_0 t)$

$$X(s) = \int_0^{\infty} \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) \times \frac{1}{2} \cancel{dt}\, e^{-st}\, dt.$$

$$= \frac{1}{2}\left\{ \int_0^{\infty} e^{j\omega_0 t} e^{-st}\, dt + \int_0^{\infty} e^{-j\omega_0 t} e^{-st}\, dt \right\}$$

$$= \frac{1}{2}\left[\frac{1}{s - j\omega_0} \times \frac{1}{s + j\omega_0}\right] = \frac{1}{2}\left[\frac{\cancel{2}s}{s^2 + \omega_0^2}\right] = \frac{s}{s^2 + \omega_0^2}$$

$$x(t) = \sin(\omega_0 t)$$

$$X(s) = \frac{\omega_0}{s^2 + \omega_0^2}$$

Properties of the Laplace Transform:-

1. Linearity:-

$$L\{a_1 x_1(t) + a_2 x_2(t)\} = a_1 X_1(s) + a_2 X_2(s)$$

eg:- $x(t) = \cos(\omega_0 t)\, u_s(t)$.

2. Convolution:-

Let $x(t) \leftrightarrow X(s)$ and $h(t) \leftrightarrow H(s)$, then

$$L\left\{\int_{0^-}^{\infty} h(t-\lambda)\, x(\lambda)\, d\lambda\right\} = H(s)\, X(s)$$

eg:- Consider the system

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s + 1/RC}$$

$$L^{-1}\{H(s)\} = h(t) = e^{-t/RC}\, u_s(t)$$

$$y(t) = L^{-1}\{Y(s)\} = \int_{0}^{\infty} e^{-(t-\lambda)/RC}\, u_s(t-\lambda)\, x(\lambda)\, d\lambda.$$

3. Integration:- Let $x(t) \leftrightarrow X(s)$

$$L\left\{\int_{0}^{t} x(\lambda)\, d\lambda\right\} = \frac{1}{s} X(s)$$

Let $x(t) = r(t)$

$$X(s) = R(s) = L\left\{\int_{0}^{t} u_s(\lambda)\, d\lambda\right\} = \frac{1}{s} \times \frac{1}{s} = \frac{1}{s^2}$$

Sample Test

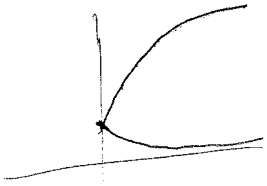
1. Draw the graph of unit step function.

2. Represent the unit triangle function.

3. Write the formulae for Sa and Sinc functions interms of Sine function.

4. Determine whether the function $\cos(\pi t)$ is even or odd

5. Sketch the function $f(t) = \begin{cases} 0, & t < 0 \\ e^{-2t} & t \geq 0 \end{cases}$



6. Define Time scaling property of signals with an example.

7. Write the procedure to ~~model~~ represent a signal defined on intervals.

8. Write the Trigonometric representation of Fourier Series.

9. Write the Computational formulae for finding the Coefficients of Cosine representation & exponential rep.

10. Find the Fourier transform of $x(t) = \Pi\left(\frac{t}{\varepsilon}\right)$.