UNIT - I

UNIT - I Covers introduction to signals, the common fors. Used to represent signals, the various operations performed on signals and Fourier series - representation of signals.

UNIT - II - Covers Fourier transforms and Laplace transforms and Inverse Laplace transforms.

UNIT - III - Discrete Time signals - Sampling, Nyquist Sampling theorem

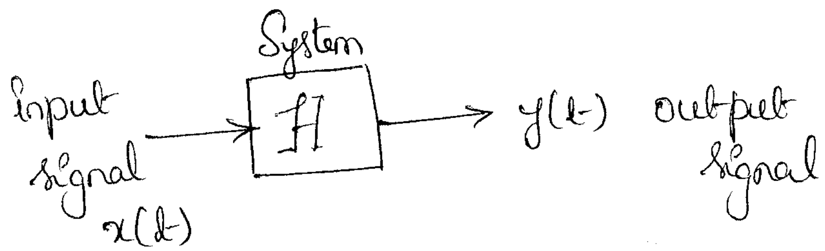
UNIT - IV - Convolution and Correlation of discrete time signals, z-transforms and Inverse z-transforms.

UNIT - V - Systems, representation, Properties of Systems, Block diagram representations, Block diagram reduction.

→ Signals and a System - The two fundamental Concepts of this subject.

→ Signal - is a mathematical description of a physical process.

eg:- speech signal, Image on a computer screen, Current, Voltage, electro magnetic waves, The movement of accelarator.



→ One signal called an input signal causes the appearance of the second signal.

→ The mathematical relation-ship between input signal and output signal is called a system.

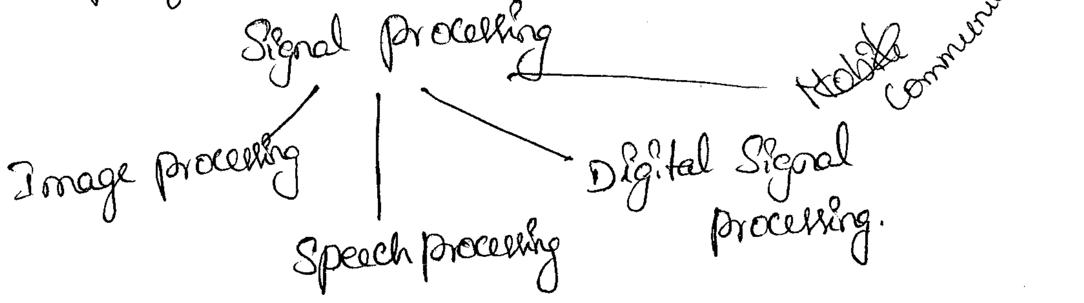
eg:- A physical process -
a device - Automobile, A stereo, A network
Interconnection of devices -

→ There are 3 components to signal theory:

Modeling - developing a mathematical description of the signal

analysis - extracting the information about the underlying physical process.

Signal design - The reverse of signal modeling and analysis.
eg:- generating a sinusoidal ~~wave~~ voltage of specific frequency and amplitude.



Continuous-Time functions

Let R be a set of real numbers

→ An interval is a set with one of the forms

$$I_1 = \{ t \in R \mid t_- < t < t_+ \} = (t_-, t_+)$$

$$I_2 = \{ t \in R \mid t_- < t \leq t_+ \} = (t_-, t_+]$$

$$I_3 = \{ t \in R \mid t_- \leq t < t_+ \} = [t_-, t_+)$$

$$I_4 = \{ t \in R \mid t_- \leq t \leq t_+ \} = [t_-, t_+]$$

I_1 is an open interval.

I_4 is a closed interval if $-\infty < t_- < t < t_+ < \infty$

I_2 and I_3 are neither open nor closed.

→ Let I_1 and I_2 be two intervals. Let 'f' be a rule which assigns a member of I_2 (t_2) to each member of I_1 ($t_1 \in I_1$); $f(t_1) = t_2$. Then we say that f is a real function.

The interval on which $f(t)$ is defined I_1 is called the domain of the function and the interval from which the function takes values I_2 is called the range.

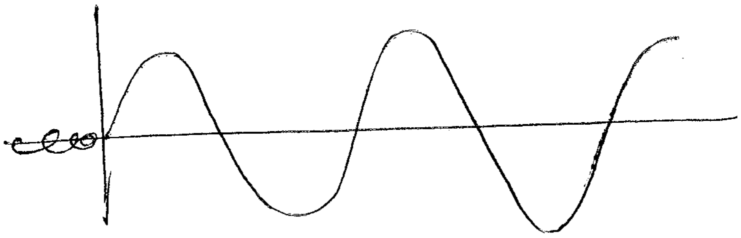
→ A fn is not defined outside its domain.

Let us consider the function $\sin t$

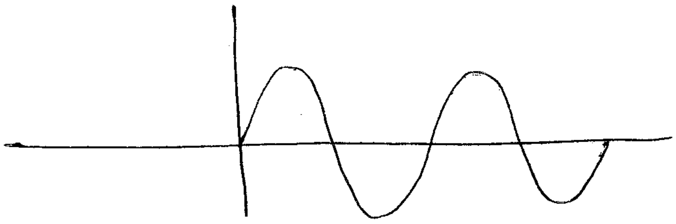
Let $I_1 = \{ t \in R \mid 0 \leq t < \infty \}$

$f_1 : I_1 \rightarrow R \qquad f_1(t) = \sin(t)$

$f_1(t) = \sin t \quad t \geq 0$



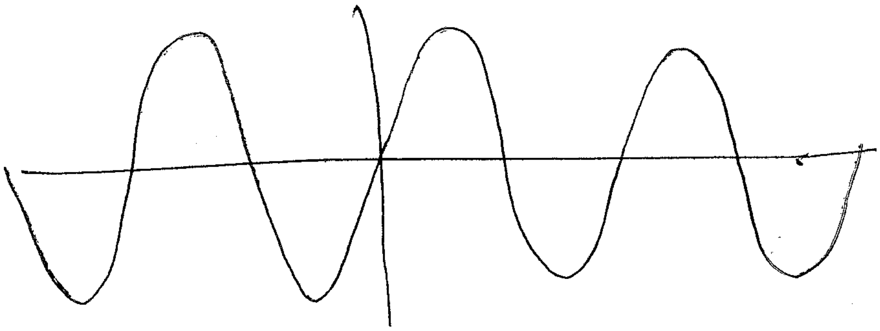
$f_2(t) = \begin{cases} \sin t & t \geq 0 \\ 0 & t < 0 \end{cases}$



$f_3(t) = \sin t$

$f_3 : R \rightarrow R$

$-\infty < t < \infty$



→ Even and odd fns:-

The fn $x(t)$ is even if $x(-t) = x(t)$

odd if $x(-t) = -x(t)$

eg:- ~~x(t)~~ $\cos t$ and $|x(t)|$ are even

$x(t) = t$ is odd.

→ If $x_o(t)$ is odd and $x_e(t)$ is even then

(a) $x_e(t)\, x_e(t)$ = even fn.

(b) $x_e(t)\, x_o(t)$ = odd fn.

(c) $x_o(t)\, x_o(t)$ = even fn.

(d) $\displaystyle\int_{-a}^{a} x_o(t)\,dt = 0$

(e) $\displaystyle\int_{-a}^{a} x_e(t)\,dt = 2\int_{0}^{a} x_e(t)\,dt.$

Common functions:-

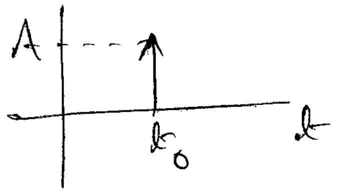
1. The unit impulse or Delta function $\delta(t)$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)$$

if $f(t)$ is continuous at $t=0$.

$\delta(t) = 0, \quad t \neq 0$

$A\,\delta(t-t_0)$ can be represented as



→ $f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$
if $f(t)$ is continuous at $t=t_0$.

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = \int_{-\infty}^{\infty} f(t_0)\,\delta(t-t_0) = f(t_0)$$

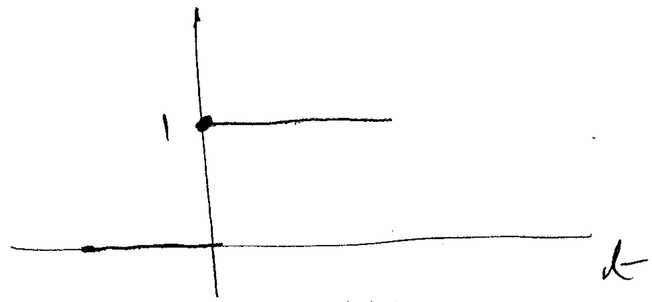
Sifting property.

2. if
$$f(t) = \begin{cases} 0 & t \leq t_1 \\ 1 & t_1 < t < t_2 \\ 0 & t_2 \leq t \end{cases}$$

$$\int_{t_1}^{t_2} f(t)\,\delta(t-t_0)\,dt = \int_{t_1}^{t_2} \delta(t_0-t_0) = \begin{cases} 1 & \text{if } t_1 < t_0 < t_2 \\ 0 & \text{if } t_0 < t_1, \; t_0 > t_2 \end{cases}$$

2. The unit step function $u_s(t)$ is defined as

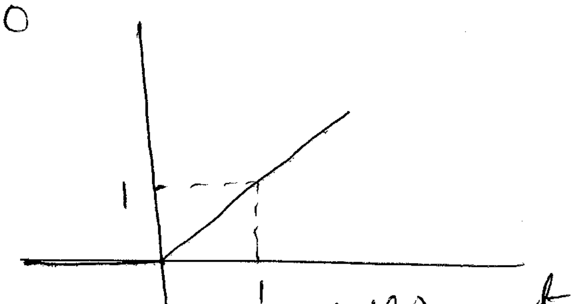
$$u_s(t) = 1, \quad t \geq 0$$
$$= 0 \quad t < 0$$



$u_s(t)$

3. The unit ramp function $r_p(t)$ is defined as

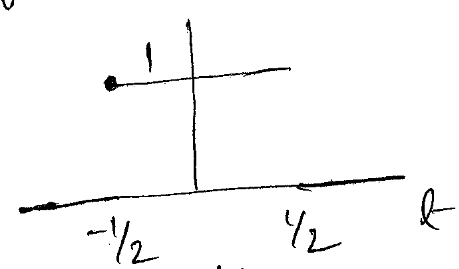
$$r_p(t) = t \quad t \geq 0$$
$$= 0 \quad t < 0$$



$r_p(t)$

4. The unit pulse function $\Pi(t)$ is defined as

$$\Pi(t) = \begin{cases} 1 & -1/2 \leq t < \frac{1}{2} \\ 0 & \text{else where} \end{cases}$$



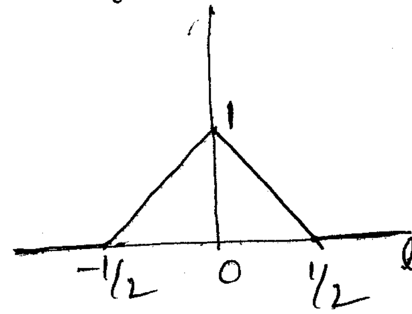
$\Pi(t)$

5. The unit triangular function $\Lambda(t)$ is defined as

$$\Lambda(t) = \begin{cases} t + 1/2, & -1/2 \leq t \leq 0 \\ 1/2 - t & 0 \leq t \leq 1/2 \\ 0 & \text{else where} \end{cases}$$

$\to 2t+1$

$\to 1-2t$



$\Lambda(t)$

$(-1/2, 0)$ $(0,1)$

$1 - 1/2\, t$

$(x + \frac{1}{2})(+1) = (y-0)(+1/2)$

$\frac{y}{2} = x + 1/2$

$y = 2x + 1$

6. The function $f(t)$ is exponential if

$$f(t) = A e^{at}$$

7. If $t > 0$, the natural logarithm of t is any number y such that $t = e^{y}$

$$y = \ln(t)$$

8. If $t > 0$, the logarithm base 10 of t is any number y such that $t = 10^{y}$

$$y = \log(t)$$

9. The function $f(t)$ is called a sinusoidal fn. if

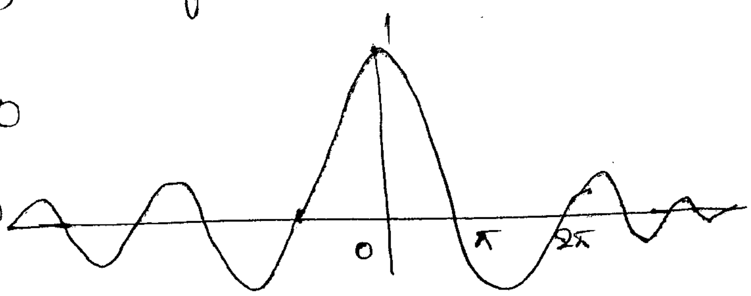
$$f(t) = A \sin(\omega_0 t + \phi)$$

A - amplitude, frequency $\underset{\text{rad/sec}}{\omega_0} = 2\pi \underset{\text{Hertz}}{f_0}$

ϕ - phase of the fn.

10. The Sa function $Sa(t)$ is defined as

$$Sa(t) = \begin{cases} \dfrac{\sin t}{t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$

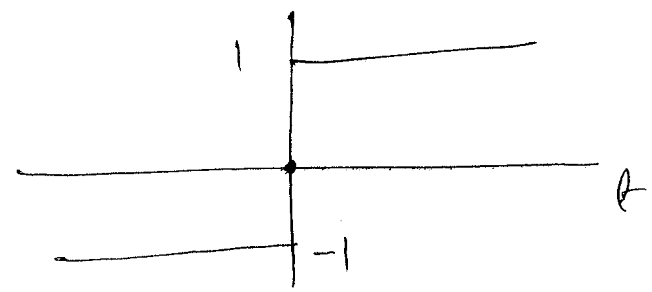


11. The Sinc function $\mathrm{Sinc}(t)$ is defined as

$$\mathrm{Sinc}(t) = \begin{cases} \dfrac{\sin \pi t}{\pi t}, & t \neq 0 \\ 1 & t = 0. \end{cases}$$

12. The sign function is defined as

$$\operatorname{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



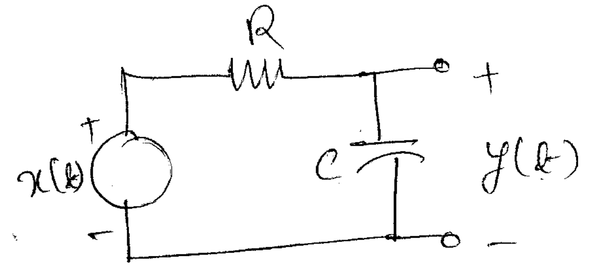
Definition of a signal :-

A signal is a function which represents the time variation of a physical variable.

A signal that depends on a real variable 't' is called a continuous time signal, and is denoted as $x(t)$

A signal that depends on a discrete variable 'n' is called a discrete - time signal and is denoted as $x(n)$.

eg:- Consider the RC network in Fig.

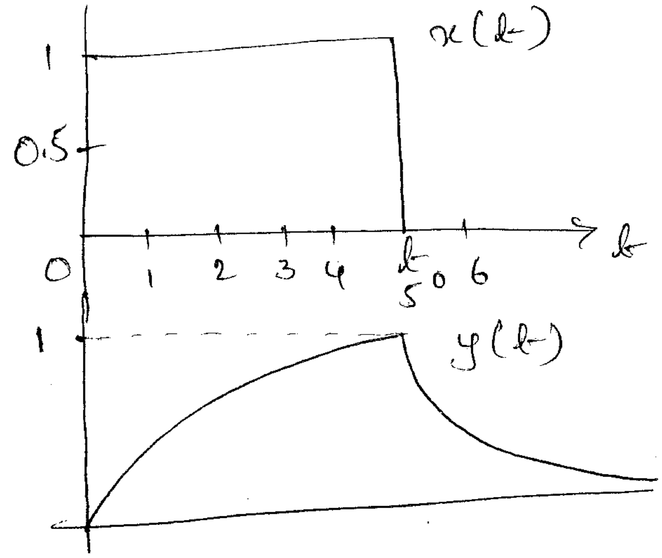


The applied voltage is a signal. It is denoted by $x(t)$
The voltage across the resistor is a signal. The current flowing through the resistor is also a signal.
Suppose at time $t=0$ we switch on the applied voltage and at time $t=1$ we switch it off. while the voltag

is on, the source applies a constant one volt to the network.

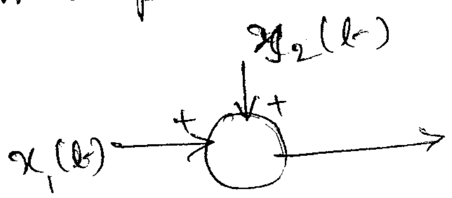
$$x(t) = u_s(t) - u_s(t-t_0)$$

$$y(t) = 1 - e^{-t/RC}, \quad 0 \le t < t_0$$
$$= e^{-\frac{t-t_0}{RC}} - e^{-t/RC}, \quad t \ge t_0$$

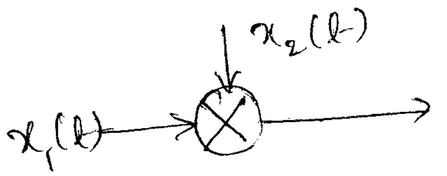


Interconnection of signals:

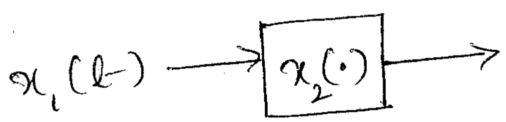
The signals can be combined in 3 ways: addition, multiplication and composition.



$x_1(t) + x_2(t) = y(t)$ Addition

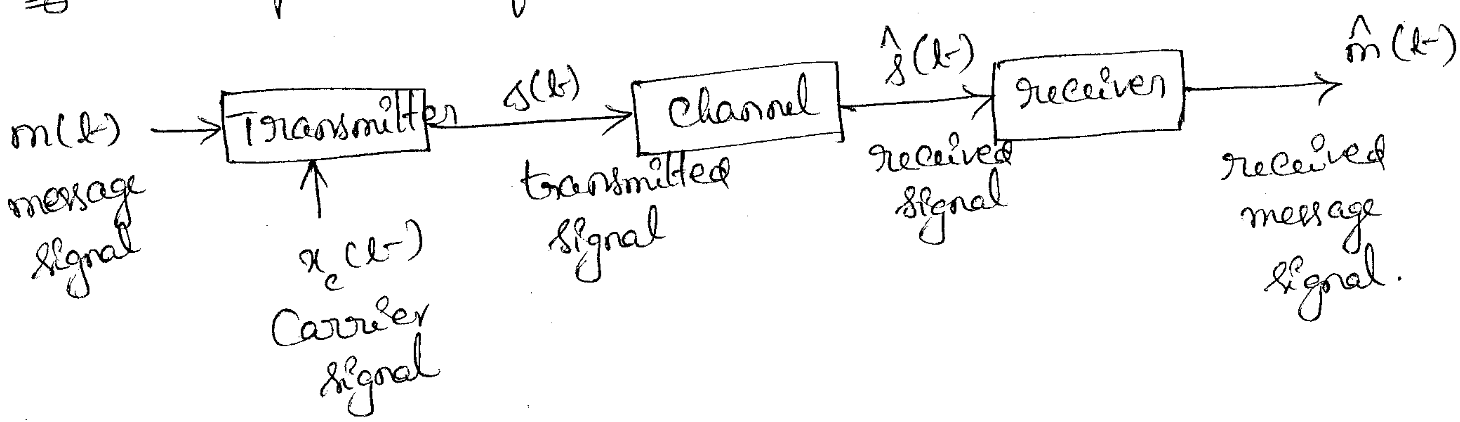


$x_1(t)\, x_2(t) = y(t)$ Multiplication

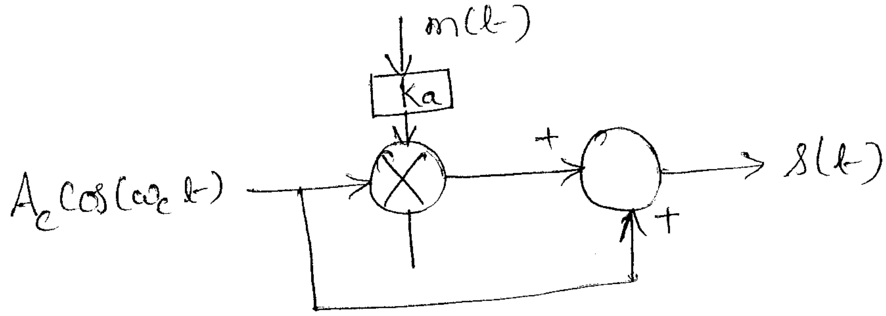


$x_2(x_1(t)) = y(t)$ Composition.

eg:- Components of a communication system:-



Let $x_c(t) = A_c \cos(\omega_c t)$



$$s(t) = A_c \cos \omega_c(t) \left[1 + K_a m(t) \right]$$

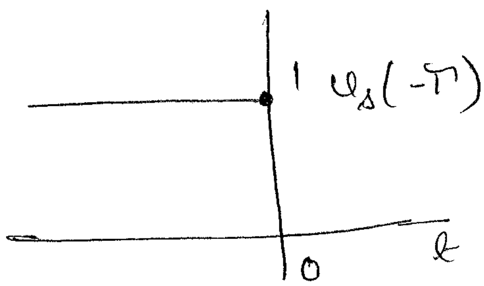
Time scaling:-

Given a signal $x(t)$, the signal $x(\varepsilon\tau) = \tilde{x}(\tau)$ is said to be time scaled.

→ if $\varepsilon = -1$, $t = -\tau$, The time scale of the signal is reversed.

eg:- $u_s(t)$

$$x(\tau) = u_s(-\tau) = \begin{cases} 1 & \tau \leq 0 \\ 0 & \tau > 0 \end{cases}$$



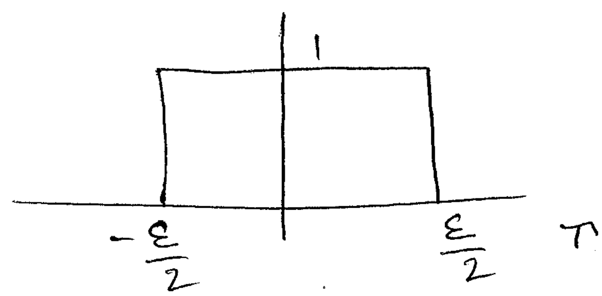
eg:- $t = \dfrac{\tau}{\varepsilon}$

$$x(\tau) = \Pi\left(\frac{\tau}{\varepsilon}\right) = \begin{cases} 1 & |\tau| \leq \frac{\varepsilon}{2} \\ 0 & |\tau| > \frac{\varepsilon}{2} \end{cases}$$

ε - pulse width.

As ε gets larger,

the pulse width expands.



As ε gets smaller, the pulse width shrinks.

Time - Shift :-

A second way to create a signal is to move the origin.

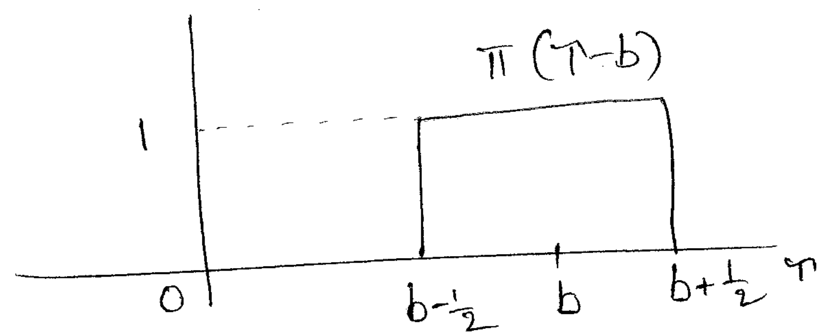
Let $t = \tau - b$

Given the signal $x(t)$, the signal $x_s(\tau) = x(\tau - b)$ has been time shifted. If $b > 0$, the time shift is a right shift.

If $b < 0$, the time shift is a left shift.

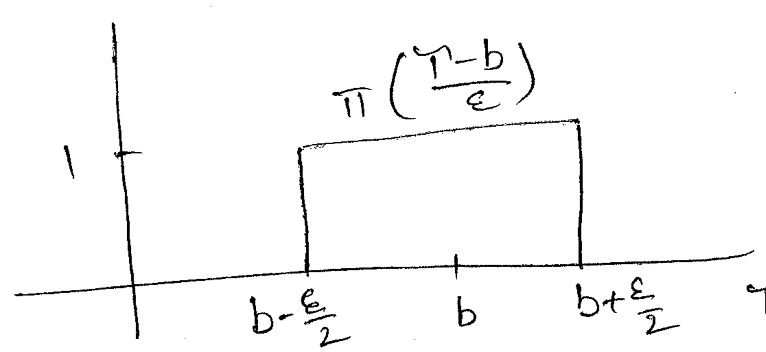
→ Consider $\Pi(\tau - b)$

The pulse is now centered at b.



→ Time shifting and Time scaling can be combined to create new functions.

$\Pi\left(\frac{\tau - b}{\varepsilon}\right)$



The width of the pulse has been changed to ε due to time scaling. It is shifted to the right so that it it is centered at $\tau = b$.

Limits of Signals :-

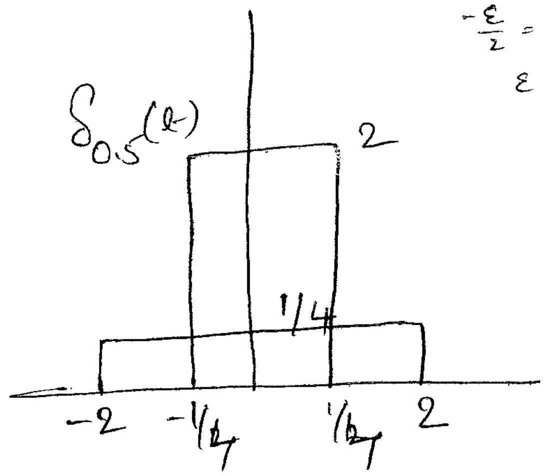
A Signal can be represented as a limit of a set of parameterized Signals.

Consider $\delta_\varepsilon(t) = \frac{1}{\varepsilon}\pi\left(\frac{t}{\varepsilon}\right), \quad \varepsilon > 0$

$-\frac{\varepsilon}{2} = -2$
$\varepsilon = 4$

$-\frac{\varepsilon}{2} = -\frac{1}{4}$
$\varepsilon = 1/2$

The signal is shown for different values of ε.



→ The limit of this sequence of signals as $\varepsilon \to 0$ is an Impulse signal.

$\delta_\varepsilon(t) = 0 \quad$ for $\quad t < -\frac{\varepsilon}{2}, \; t > \frac{\varepsilon}{2}$

$$\int_{-\infty}^{\infty} \delta_\varepsilon(t) = 1, \quad \varepsilon > 0$$

Because the area is constrained to be one, the height of the signal tends to be infinity as $\varepsilon \to 0$

$$\lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} \delta_\varepsilon(t)\, x(t)\, dt = \int_{-\infty}^{\infty} \delta(t)\, x(t)\, dt = x(0)$$

$$\therefore \lim_{\varepsilon \to 0} \delta_\varepsilon(t) = \delta(t)$$

Signals defined on intervals:-

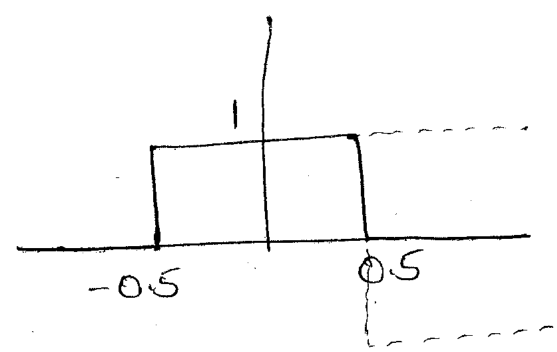
→ A complex signal can be represented in terms of simple signals, defined on intervals.

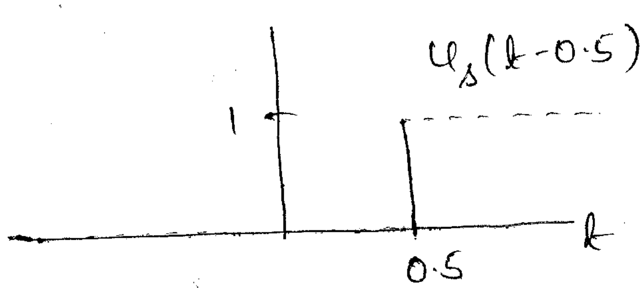
→ The signal modeling proceeds by constructing a signal model on each interval and finally these models are combined to complete the signal model.

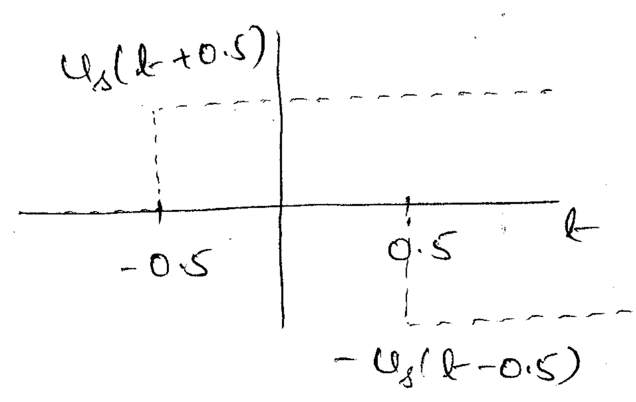
→ The representation of a signal on an interval has two components: (1) a pulse that selects an interval
(2) a signal representation for that interval.

eg:- Consider the pulse shown below: It can also be represented using the unit pulse or unit step functions:

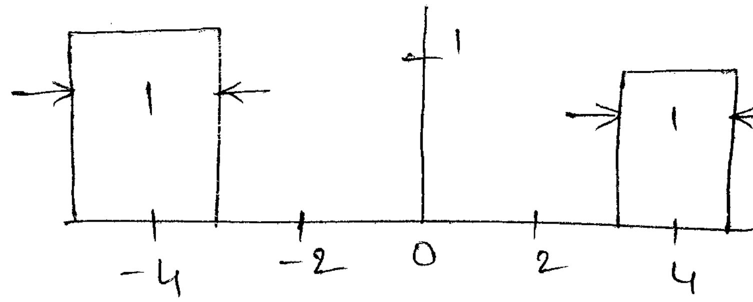
$$\therefore \Pi(t) = u_s(t+\tfrac{1}{2}) - u_s(t-1/2)$$







eg:- Construct a model of the following signal:



For the pulse centered at $t=-4$

$$x_-(t) = \Pi(t+4)$$

$$x_+(t) = \Pi(t-4)$$

$$x(t) = x_-(t) + x_+(t)$$

Procedure to model a signal:-

Let $x(t)$ be the given signal.

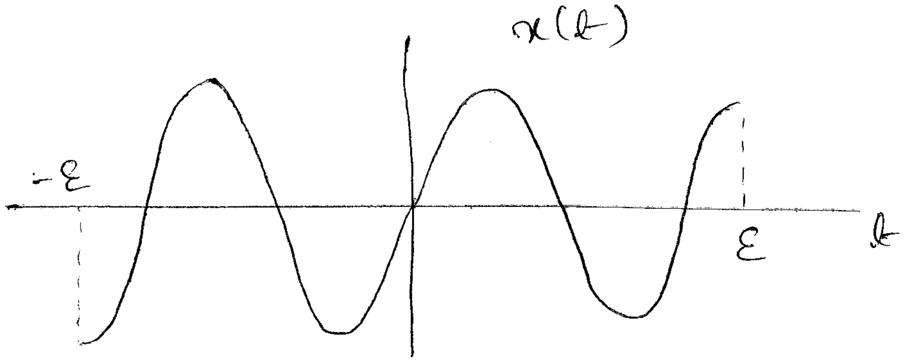
1. Divide the signal into intervals, such that the signal on each interval has a simple representation.

2. Write the pulse function for each interval. This fn. is one over the interval of interest and zero elsewhere.

3. Develop a signal representation on each interval. Multiply this representation by the appropriate pulse function.

4. Sum the representations for each model interval to form the signal model.

eg:- Truncated Sinusoid



The only one interval of Interest

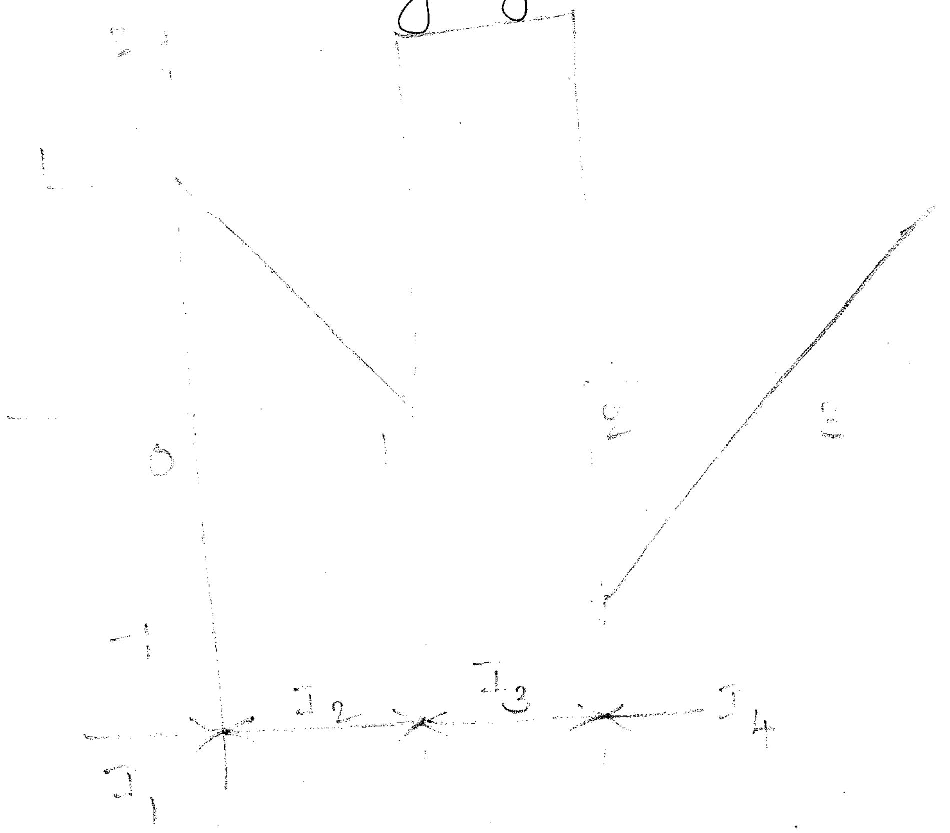
$$I_1 = \{ t \mid -\varepsilon \leq t < \varepsilon \}$$

The signal representation over the interval I_1 is $\sin(\omega t)$

$$\Pi(t) = u_s(t+\varepsilon) - u_s(t-\varepsilon)$$

$$x(t) = \sin(\omega t)\left[u_s(t+\varepsilon) - u_s(t-\varepsilon)\right].$$

eg:- Model the following signal



1. Break the signal into the intervals

$I_1 = \{ t \mid t < 0 \}$

$I_2 = \{ t \mid 0 \le t < 1 \}$

$I_3 = \{ t \mid 1 \le t < 2 \}$

$I_4 = \{ t \mid t \ge 2 \}$

2. Construct a pulse function for each interval

for I_1 $\quad 1 - u_s(t)$

for I_2 $\quad u_s(t) - u_s(t-1)$

for I_3 $\quad u_s(t-1) - u_s(t-2)$

for I_4 $\quad u_s(t-2)$

3. * For interval I_1, the function is a constant value of 1.

$\therefore x(t)_{I_1} = 1\,[1 - u_s(t)] = [1 - u_s(t)]$

* For interval I_2, the signal is $(1-t)$

$\therefore x(t)_{I_2} = (1-t)\,[u_s(t) - u_s(t-1)]$

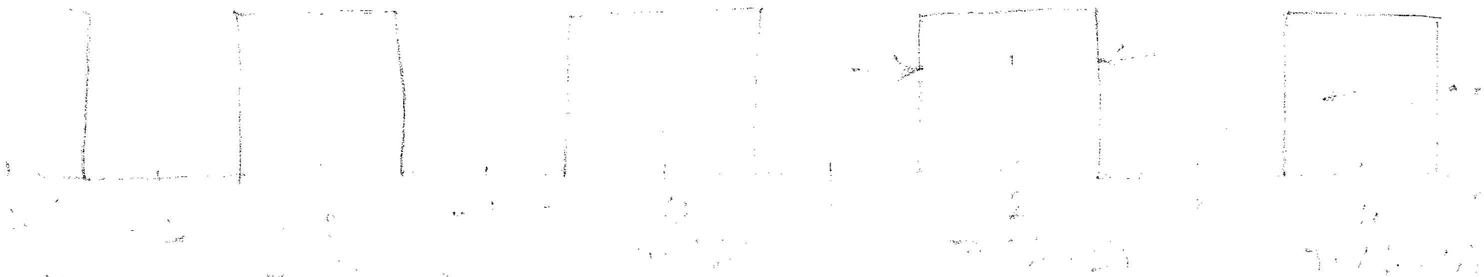
* For interval I_3, the signal is 2

$\therefore x(t)_{I_3} = 2\,[u_s(t-1) - u_s(t-2)]$

For interval I_4 $(t-3)\left[u_s(t-2)\right] = x(t)_{t_4}$

$$x(t) = \left[1-u_s(t)\right] + \left[u_s(t)-u_s(t-1)\right](1-t) + 2\left[u_s(t-1)-u_s(t-2)\right] + (t-3)\left[u_s(t-2)\right]$$

eg:- Find the representation of the following signal:



This signal is a string of shifted pulses. It can be represented by summing up all the pulses.

$$x(t) = \sum_{m=-\infty}^{\infty} \Pi(t-2m)$$

Pulse trains are used to illustrate many concepts of signal analysis.

Signals as sums of Sinusoids

→ This is another approach of signal representation.

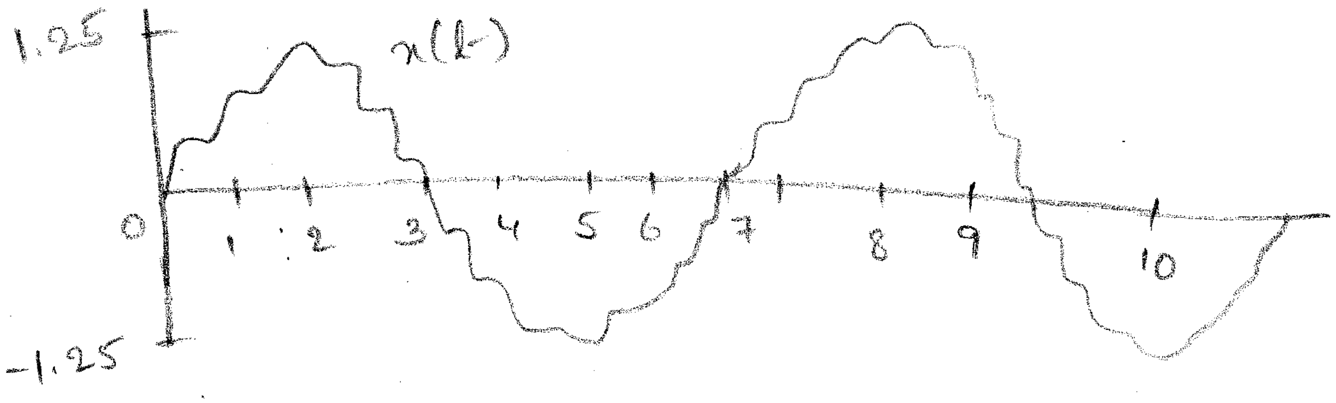
The basic building block of this type of signal representation is the sinusoid.

→ A single sinusoid is defined by a frequency, an amplitude and a phase.

Complicated signals are represented by sums of sinusoids with different frequencies, amplitudes and phases. Each sinusoid is defined over the entire time axis rather than over a just finite interval.

This representation plays a fundamental role in signal modeling.

eg:- The following graph of a voltage measured in the lab.



The period T_1 of the low frequency Component is

$$T_1 = 6.2 = \frac{2\pi}{\omega_1} \text{ sec} \quad (\text{or}) \quad \omega_1 \simeq 1 \text{ rad/sec.}$$

The amplitude of the low frequency sinusoid is 10 ~~cycles~~

$A \approx 1$

The high frequency sinusoid has approximately 10 cycles per cycle of the low frequency component.

$\therefore \omega_2 = 10\omega_1 = 10$ rad/sec.

Amplitude of high frequency component $B \approx 0.1$

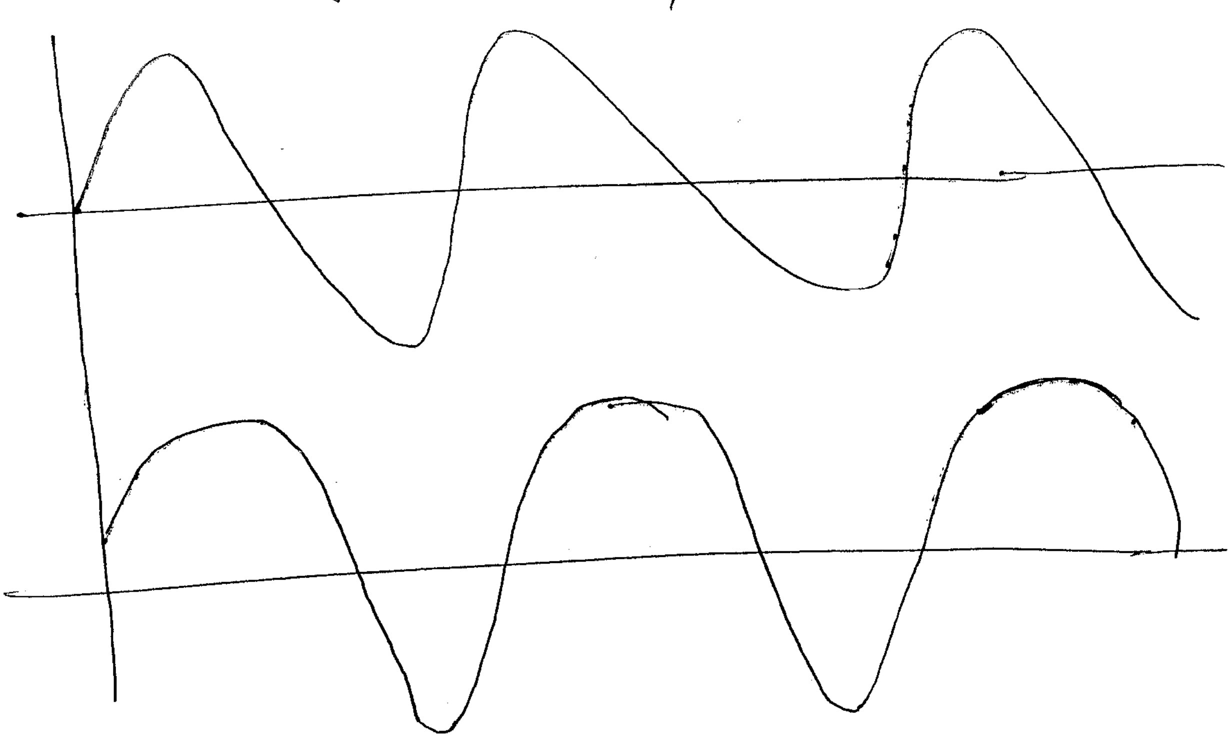
$x(t) = 0$ at $t = 0$, $\therefore \theta = 0$

$x(t) = (1 \sin t) + (0.1 \sin 10t)$

→ By varying the parameters of the sinusoids, the signal shape can be varied significantly.

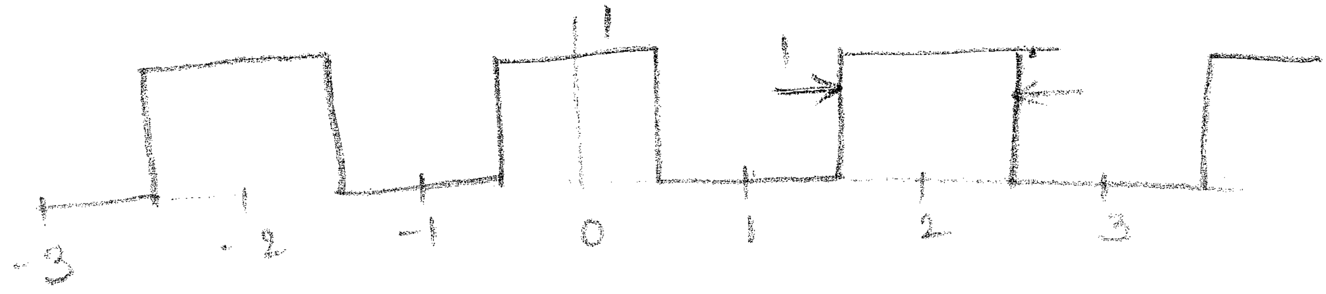
eg:- $x(t) = \sin(t) + 0.2 \sin(2t + \theta)$

for $\theta = 0$, $\theta = 90$



So, far we have considered a finite sum of sinusoids. By increasing the no. of sinusoids to '∞' as long as the parameters of sinusoids satisfy certain conditions, we can model complicated signals. These infinite sums of sinusoids are called Fourier Series.

eg:- Consider the periodic signal



The Fourier series for this signal is given by

$$x(t) = \frac{1}{2} + \sum_{\substack{m=1 \\ m\,\text{odd}}}^{\infty} \frac{2}{m\pi} \cos(m\pi t + \theta_m) = \sum_{k=-\infty}^{\infty} \Pi(t-2k)$$

where $\theta_m = \begin{cases} \pi, & m = 3, 7, 11, \cdots \\ 0, & m = 1, 5, 9, \cdots \end{cases}$

Fourier Series:-

Periodic signals:-

The signal $x(t)$ is periodic, if there exists a constant $T_0 > 0$ such that

$$x(t+T_0) = x(t) \quad \text{for all } t.$$

The smallest T_0 for which this is true is called the fundamental period. All other signals are aperiodic.

$x_{T_0}(t)$ - periodic signal with period T_0.

eg:- The pulse train is a periodic signal with a period $T_0 = 2$.

$\sin(t)$ and $\cos(t)$ are periodic signals with period $T_0 = 2\pi$

→ Let $x_1(t)$ be an aperiodic signal defined on the interval $t_0 \le t \le t_0 + T_0$. Then define a signal $x_{T_0}(t)$ by repeating the signal $x_1(t)$ on all the intervals $mt_0 \le t \le mt_0 + T_0$, where m is an integer $\neq 0$.

The signal $x_{T_0}(t)$ is said to be created by periodic extension.

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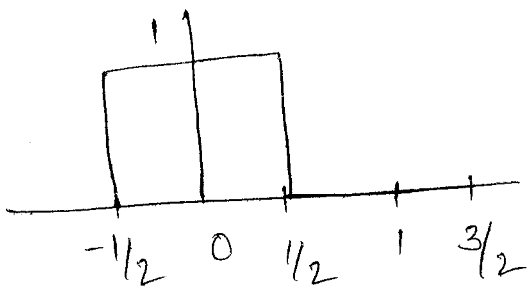
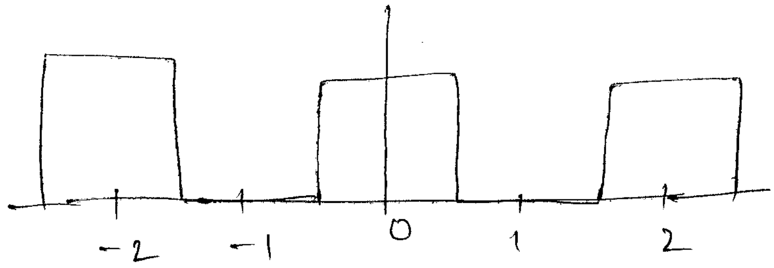
eg:- Let the signal definition

$$x_1(t) = \begin{cases} 1, & -\frac{1}{2} \le t < \frac{1}{2} \\ 0, & \frac{1}{2} \le t < \frac{3}{2} \end{cases}$$



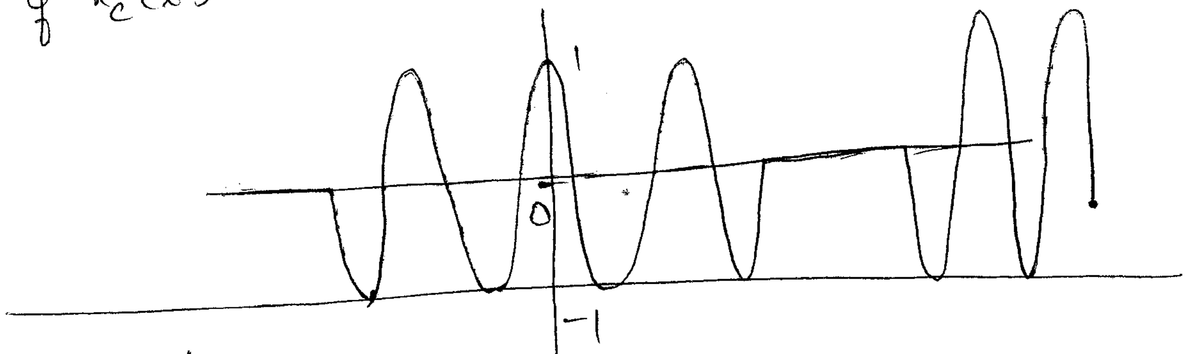


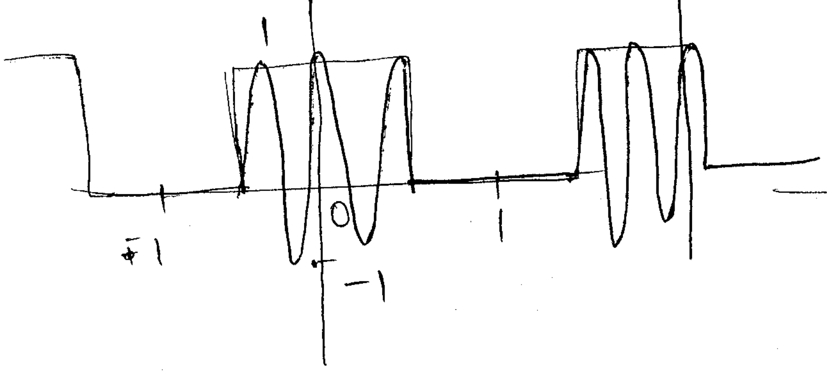
The periodic extension of the signal is the pulse train.

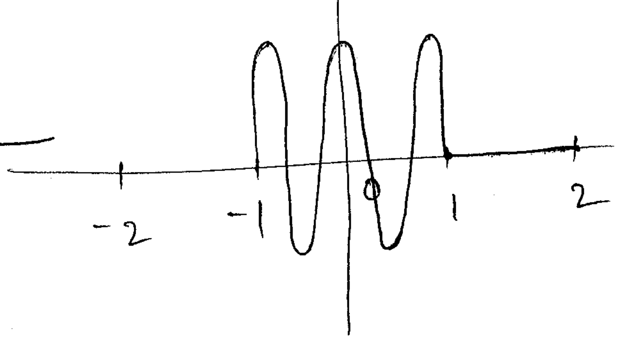
→ Define a new signal $x_c(t)$ by multiplying $x_1(t)$ by $\cos(\omega_c t)$

$$x_c(t) = \begin{cases} \cos(\omega_c t), & -1/2 \le t < 1/2 \\ 0, & 1/2 \le t < 3/2 \end{cases}$$

We can define a new signal $x_p(t)$ by periodic extension of $x_c(t)$.







Three Representations of Fourier Series:-

1. Cosine Representation
2. Trigonometric Representation
3. Exponential Representation.

1. Cosine Representation:-

Def:- Let $x(t)$ be a periodic signal $(-\infty < t < \infty)$ with period T_0, If there exists a Convergent series of the form

$$x(t) = \sum_{m=0}^{\infty} A_m \cos(m\omega_0 t + \theta_m), \quad \omega_0 = \frac{2\pi}{T_0}, \; A_m > 0,$$

then this series is called a Cosine Fourier series.

The numbers A_m are Called the one-sided amplitude coefficients.

θ_m - One sided phase Coefficients.

Features (a) All terms are Cosine functions.

(b) The amplitude Coefficients are +ve. $A_m \geq 0$, and each term Contains a phase angle θ_m.

(c) The summation index ranges from 0 to ∞.

(d) There is no easy formula to calculate this form of the Fourier series.

It Can be determined from one of the other representations.

2. Trigonometric Representation :-

$$A_m \cos(m\omega_0 t + \theta_m) = A_m(\cos\theta_m)\cos(m\omega_0 t) + (-A_m \sin\theta_m)\sin(m\omega_0 t)$$

$$= a_m \cos(m\omega_0 t) + b_m \sin(m\omega_0 t)$$

Def:- If $x(t)$, $-\infty < t < \infty$, is a periodic signal with fundamental period T_0. If there exists a convergent series of the form

$$x(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos(m\omega_0 t) + \sum_{m=1}^{\infty} b_m \sin(m\omega_0 t),$$

$\omega_0 = \frac{2\pi}{T_0}$, then this series is called a Trigonometric Fourier series.

Features:-

1. The series contains both sine and cosine terms.
2. The amplitude coefficients can be both +ve and -ve. There are no phase angles associated with these terms.
3. m ranges from 0 to ∞.

Cosine vs Trigonometric:-

$$\left.\begin{aligned} a_0 &= A_0 \cos\theta_0 \\ a_m &= A_m \cos\theta_m \\ b_m &= -A_m \sin\theta_m \end{aligned}\right\} \text{ Cosine to Trigonometric.}$$

$a_m^2 + b_m^2 = A_m^2$

$A_m = \sqrt{a_m + b_m}$, $\theta_m = -\tan^{-1}\left(\frac{b_m}{a_m}\right) \rightarrow$ Trigonometric to Cosine.

$\frac{b_m}{a_m} = -\tan\theta_m$

3. Exponential Representation:-

$$A_m \cos(m\omega_0 t + \theta_m) = A_m \frac{e^{j(m\omega_0 t + \theta_m)} + e^{-j(m\omega_0 t + \theta_m)}}{2}$$

(Eulers identity)

$$x(t) = A_0 + \sum_{m=1}^{\infty} \frac{A_m}{2} e^{j\theta_m} e^{jm\omega_0 t} + \sum_{m=1}^{\infty} \frac{A_m}{2} e^{-j\theta_m} e^{-jm\omega_0 t}$$

$$= A_0 + \sum_{m=1}^{\infty} \left(\frac{A_m e^{j\theta_m}}{2}\right) e^{jm\omega_0 t} + \sum_{m=1}^{\infty} \frac{A_m e^{j(-\theta_m)}}{2} e^{-jm\omega_0 t}$$

$$= A_0 + \sum_{m=1}^{\infty} \left(\frac{A_m e^{j\theta_m}}{2}\right) e^{jm\omega_0 t} + \sum_{k=-1}^{\infty} \frac{A_k e^{j\theta_k}}{2} e^{jk\omega_0 t}$$

$A_m = A_k$, $(-\theta_m)_{m>0} = (\theta_k)_{k<0}$

Let $X_0 = A_0$, $X_m = \frac{A_m}{2} e^{j\theta_m}$, $m > 0$

$$x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jm\omega_0 t}$$

Def :- If $x(t)$, $-\infty < t < \infty$, is a periodic function with fundamental period T_0, if there exists a convergent series of the form

$$x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jm\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T_0}$$

This series is called the exponential Fourier series.

$|X_m|$ are called the two-sided amplitude coeff.

$\angle X_m$ – two sided phase coefficients.

Exponential Vs Cosine

$$X_m = \frac{A_m}{2} \left(\cos\theta_m + j \sin\theta_m \right)$$

Exponential Vs Trigonometric.

$$X_m = \frac{1}{2} (a_m - j b_m)$$

Existence theorem:-

Let $x(t)$, $-\infty < t < \infty$, be a periodic signal with period T_0, Assume that

(a) $x(t)$ is absolutely integrable over one period

$$\int_{T_0} |x(t)|\, dt < \infty$$

(b) The signal $x(t)$ has only a finite number of minima and maxima over any period and

(c) the signal $x(t)$ has only a finite number of discontinuities, over any period.

Then the Fourier series representation

$$x(t) = \sum_{m=0}^{\infty} A_m \cos(m\omega_0 t + \theta_m) \quad , \quad \omega_0 = \frac{2\pi}{T_0}$$

exists and is unique.

Problem 1:- Translate the following Cosine series into trigonometric series.

$$x(t) = \frac{1}{2} + \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{2}{m\pi} \cos(m\pi t + \theta_m)$$

where
$$\theta_m = \begin{cases} -\pi, & m = 3, 7, 11, \ldots \\ 0, & m = 1, 5, 9, \ldots \end{cases}$$

$$a_0 = A_0 \cos \theta_0 = \frac{1}{2}$$

$$a_m = A_m \cos \theta_m = \frac{2}{m\pi} \cos(-\pi), \quad m = 3, 7, 11, \ldots$$
$$= \frac{2}{m\pi} \cos(0), \quad m = 1, 5, 9, \ldots$$

$$a_m = (-1)^{\frac{m-1}{2}} \frac{2}{m\pi}, \quad m \text{ odd}.$$

$$b_m = -A_m \sin \theta_m = \frac{2}{m\pi} \sin(-\pi), \quad m = 3, 7, 11, \ldots$$
$$= \frac{2}{m\pi} \sin(0), \quad m = 1, 5, 9 \ldots$$

$$= 0.$$

$$\therefore x(t) = \frac{1}{2} + \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} (-1)^{\frac{m-1}{2}} \frac{2}{m\pi} \cos(m\pi t)$$